the constant-pressure specific heat, $C_{p},{ }^{13}$ derived from measurements on the saturated liquid. If the relations hold at higher pressures, then for the limits $5 \times 10^{-2}$ $<\left|P-P_{\lambda}\right|<10 \mathrm{~atm}$ or $5 \times 10^{-4}<\left|T-T_{\lambda}\right|<10^{-1}{ }^{\circ} \mathrm{K}, \beta$ tends to vary linearly with $\alpha_{P}$ and $C_{p}$, which is consistent with the Buckingham-Fairbank ${ }^{14}$ derivations. Unfortunately, the experimental ranges of pressure do not overlap. Therefore direct comparisons between the data cannot be made. However, at the $\lambda$ point of $2.023^{\circ} \mathrm{K}$ and 13.04 atm Lounasmaa ${ }^{4}$ found that $\beta$, measured with $10^{-3} \mathrm{~atm}$ resolution, varied linearly with $\left|P-P_{\lambda}\right|$ for $10^{-3}<\left|P-P_{\lambda}\right|<10^{-2} \mathrm{~atm}$. At $\left|P-P_{\lambda}\right|=10^{-3} \mathrm{~atm}$, his results coincide with the values from Eqs. (3) and (5), namely, $\beta_{-}=8.8$ and $\beta_{+}=7.9$ in $10^{-3} \mathrm{~atm}^{-1}$ units. At $\left|P-P_{\lambda}\right|=10^{-2} \mathrm{~atm}$, the agreement is poorer but still acceptable. It is notable that the highest values of $\beta$ observed near a $\lambda$ point are only $\sim 10^{-2} \mathrm{~atm}^{-1}$.

Table II. Constants in Eq. (5).

| $T$ <br> $\left({ }^{\circ} \mathrm{K}\right)$ | $P_{\lambda}$ <br> $(\mathrm{atm})$ | $a_{-}$ <br> $\left(\mathrm{atm}^{-1}\right)$ | $b_{-}$ <br> $\left(\mathrm{atm}^{-1}\right)$ | $a_{+}$ <br> $\left(\mathrm{atm}^{-1}\right)$ | $b_{+}$ <br> $\left(\mathrm{atm}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.050 | 10.92 | 0.75 | 0.42 | 0.16 | 0.34 |
| 2.000 | 14.62 | 0.93 | 0.48 | 0.20 | 0.41 |
| 1.949 | 18.27 | 1.08 | 0.58 | 0.25 | 0.46 |
| 1.899 | 21.65 | 1.38 | 0.74 | 0.55 | 0.53 |
| 1.880 | 22.86 | 1.52 | 0.77 | 0.55 | 0.44 |
| 1.865 | 23.81 | 1.78 | 0.92 | 0.58 | 0.53 |
| 1.799 | 27.74 | 1.79 | 1.23 | 0.62 | 0.60 |

Therefore, the validity of an expression like Eq. (5) cannot continue indefinitely as the $\lambda$ point is approached. Goldstein ${ }^{15}$ pointed out that the root-meansquare temperature fluctuations of the system, the upper limit of meaningful $\left|T-T_{\lambda}\right|$ values, is $\sim 10^{-12^{\circ}} \mathrm{K}$.

The sound velocities of Atkins and Stasior ${ }^{2}$ were combined with the densities of Keesom and Keesom ${ }^{1}$ to derive the adiabatic compressibilities, $\beta_{S}=\left(\rho u^{2}\right)^{-1}$. Although the velocities should have high resolution, no anomalous variation of $\beta_{S}$ with pressure was seen near

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Fig. 9. The ratio of specific heats $C_{P} / C_{V}$ versus pressure for liquid $\mathrm{He}^{4}$ at several temperatures.
the $\lambda$ transition. The $\beta_{S}$ values were combined with the present isothermal compressibilities to derive $C_{P} / C_{V}$ $=\beta / \beta_{S}$, the ratio of specific heats. Figure 9 shows $C_{P} / C_{V}$ rising with pressure, reaching peaks of $\sim 1.6$ at the $\lambda$ transition, before dropping to the values at $2.20^{\circ} \mathrm{K}$, which are at most 1.05 . The peak heights of the $C_{P} / C_{V}$ ratio are indefinite, as are those of $\beta$, whereas the derivations of Buckingham and Fairbank ${ }^{14}$ indicate that if $C_{P} \rightarrow \infty, \beta \rightarrow \infty$ while $C_{V}$ and $\beta_{S}$ remain finite. However, this behavior of $C_{V}$ and $\beta_{S}$ can be questioned if the $\lambda$ transition is connected with the liquid-gas critical point [see Tisza ${ }^{16}$ and Green ${ }^{17}$ ]. As the critical point is approached, singular functions are indicated for $\beta_{S}$ and $C_{V}$ by Chase, Williamson, and Tisza ${ }^{18}$ and by Moldover and Little, ${ }^{19}$ respectively. Therefore, the functions for $\beta_{S}$ and $C_{V}$ might be similar enough to those for $\beta$ and $C_{P}$ that $C_{P} / C_{V}=\beta / \beta_{S}$ remains finite at the $\lambda$ transition.

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